

OPTIMAL IDENTIFICATION OF MULTIDIMENSIONAL POLYNOMIALS DESCRIBING SYSTEMATIC ERRORS IN CNC MACHINES

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Abstract

Systematic closed loop geometric errors in CNC machines have three translational and three rotational components for each machine slide. These components can be conveniently approximated by multidimensional polynomials that are functions of the CNC machine axes and have a particular structure. Their identification allows to compute corrections that make the CNC machine more accurate. Since measurements are in general expensive and time consuming it is important to keep their number as low as possible. In this paper optimal sampling properties for some classes of multidimensional polynomials relevant to the CNC machine closed loop error correction are presented. The optimality criteria considered in this paper are the reduction of the estimate uncertainties and of the worst case prediction error that can be committed when using the estimated functions. All results are derived in the case in which measurement errors and uncertainties are characterized by tolerances.

Keywords:

Optimal experiment, error reduction, identification, systematic error correction.

1 INTRODUCTION

The accuracy of CNC machines is affected by several factors that include geometric errors, dynamic effects of the forces applied in the milling process, thermal deformations etc. [1], [2], [3] Such errors are usually corrected measuring, for each axis of the CNC machine, the six elementary error components (three linear errors and three rotational ones) that can be associated to the movements of any rigid body. The hardware of the machine is then corrected accordingly, or the corrections are embedded in the software of the controller. A recent overview on geometric error identification and compensation can be found in [4]. In [5] a different approach is outlined. It consists in focusing on the total closed loop volumetric error that accounts for the position and orientation error of the tool with respect to the work piece within the working space of the machine. It has, in general, three linear error components of the position of the cutting tool (dx , dy , dz) and three angular components of its orientation (di , dj , dk). Total error components are functions of the n joint coordinates of the CNC machine, since they are the result of the combined effect of the elementary error components associated with the movement of each CNC machine axis. In [5] the general framework for deriving such relations is presented and the specific functions relating total closed loop volumetric errors to the elementary error components are derived for one of the most common structure of five axes CNC machine.

To provide a mathematical description for the errors, which depend on the imprecision of the CNC machine production process and do not have any particular model structure, it is usually assumed that elementary error components are adequately represented by monodimensional polynomials of the relevant CNC joint. It follows, therefore, that total closed loop volumetric errors are n -dimensional polynomials which are functions of the n joint coordinates of the CNC machine and deserve a particular structure. Specifically the angular error components are the sum of n monodimensional polynomials each one of which is the function of a different CNC joint coordinate. The degree of such polynomials is the same as the degree of the polynomials used to describe the elementary error

components. The position error components of total closed loop volumetric error either have the same structure as the angular components, or they have also additional extra terms which, however, are the result of the multiplication of a monodimensional polynomial in one of the joint coordinates by a first degree monomial in one of the joint coordinates. All the monomial of such polynomial can therefore be function of at most two joint coordinates one of which must be at power 1.

In order to compensate and correct the CNC machine it is necessary to identify the total closed loop volumetric errors that needs therefore to be measured in convenient locations within the workspace of the machine. There are different possible approaches for the measurement of such errors which make use of suitable equipment like the laser tracker or the one which is the base for the procedure patented by Jae, Hee, and Nam [6]. Alternatively total volumetric errors can be derived measuring the differences between nominal and actual values of the dimensions in a convenient artifact which has been milled with the machine to be calibrated. This last approach is described in [7] and [8].

In any case measurements are in general expensive in terms of time and equipment and it is therefore advisable to reduce their number possibly keeping it to a minimum. On the other hand accurate identification is needed to ensure good error correction. Since, however the "goodness" of the identification is related to the number and the location of the available measurements, an optimal experiment design aimed at ensuring the best possible error identification should be performed. This mainly consists in an optimal sampling selection that ensures the largest reduction of the identification error with a given number of measurements.

In this paper the problem of optimal sampling selection is considered for multidimensional polynomials of the particular structures that total closed loop error components in CNC machines can assume.

2 OPTIMAL SAMPLING SELECTION

Optimal experiment design and optimal sampling selection have been studied in different fields and a general overview of the subject is by far out of the scope of this paper. Here attention is focused only on optimal sampling design for polynomials and some results that can be found in literature for monodimensional polynomials are shortly revised as introductory background.

2.1 Problem statement

Let $P(t, \theta)$ be a polynomial represented in the Vandermonde base so that

$$P(t, \theta) = \theta_0 + \theta_1 t + \theta_2 t^2 + \dots + \theta_{p-1} t^{p-1} \quad (1)$$

The parameter vector $\theta = [\theta_0, \theta_1, \theta_2, \dots, \theta_{p-1}]$ needs to be identified using suitable measurements collected according to a sampling schedule which is a set $T_m = \{\tau_1, \tau_2, \tau_3, \dots, \tau_m\}$ of m sampling times that belong to a given interval $[t_a, t_b]$ so that, without loss in generality, $t_a \leq \tau_1 \leq \tau_2 \leq \dots \leq \tau_m \leq t_b$. Indeed the parameter vector can be identified only if $m \geq p$ and at least m measurements are collected at different times. In such condition, if measurements were error free, the parameters could be exactly derived. Since however there is always a measurement error $e(t)$, only perturbed measurements

$$y(t) = P(t, \theta) + e(t) \quad (2)$$

can be collected. Therefore parameter estimates can be derived only with some degree of uncertainty. The aim of optimal experiment design is to find out the best choice of sampling times τ_i , $i = 1, \dots, m$ that ensures some optimality of the derived parameter estimates.

To solve the problem it is necessary to define how the measurement error $e(t)$ is characterized and to state the optimality criterion used to evaluate the goodness of the estimates. Here results are reported for the case in which a set membership description is provided for the measurement error $e(t)$ that is therefore assumed to belong to a given set $\Omega_e(t)$. With this assumption no statistical characterization of error $e(t)$ is needed, although it not excluded that the error could be also statistically described. In particular the case of unknown but bounded errors is considered here, in which

$$|e(t)| \leq E. \quad (3)$$

Note that this environment fits perfectly in the framework in which CNC machines are operated, since relation (3) mainly assumes that measurements are derived with a given tolerance E .

In such condition when a vector of m measurements $Y = [y(\tau_1), y(\tau_2), y(\tau_3), \dots, y(\tau_m)]$ is collected at the T_m sampling times the following relation holds

$$Y = A(T_m)\theta + e \quad (4)$$

in which $Y \in R^m$, $\theta \in R^p$, and $e \in R^m$ is the (unknown) vector of the measurement error realization that belongs to the measurement error set

$$\Omega_e = \{e \in R^m; |e_i| \leq E, i = 1, \dots, m\} \quad (5)$$

$A(T_m)$ is the Vandermonde matrix

$$A(T_m) = \begin{bmatrix} 1 & \tau_1 & \tau_1^2 & \dots & \tau_1^{p-1} \\ 1 & \tau_2 & \tau_2^2 & \dots & \tau_2^{p-1} \\ 1 & \tau_3 & \tau_3^2 & \dots & \tau_3^{p-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \tau_m & \tau_m^2 & \dots & \tau_m^{p-1} \end{bmatrix}. \quad (6)$$

The size of the parameter feasible set

$$D_\theta(T_m) = \{\theta \in R^p; Y - A(T_m)\theta = \varepsilon; \varepsilon \in \Omega_e\}. \quad (7)$$

which is the set of all parameters which are consistent with the measurements, the polynomial model and the measurement error characterization describes the uncertainty on the parameter estimates. Any point in $D_\theta(T_m)$ could have generated the data and could therefore be used as parameter estimate, although the most convenient choice is to use its center θ^o defined as

$$\theta^o = \sup_{\theta \in D_\theta(T_m)} \|\theta^o - \theta\|_\infty = \inf_{\theta \in R^p} \sup_{\theta \in D_\theta(T_m)} \|\theta - \theta\|_\infty. \quad (8)$$

The set $D_\theta(T_m)$ depends on the sampling schedule T_m as well as on the unknown error realization e . It is however possible to compute the worst case parameter feasible set

$$\tilde{D}_\theta(T_m) = \{\theta \in R^p; A(T_m)\theta = \varepsilon; \varepsilon \in \Omega_e\} \quad (8)$$

whose size is equal to the largest size that the parameter feasible set $D_\theta(T_m)$ can have in connection with the worst error realization. To compute $\tilde{D}_\theta(T_m)$ no actual measurements are needed, but only the sampling schedule T_m is required.

For what concerns the optimality criterion to be used in the selection of the optimal sampling design two choices are mainly possible:

- When the focus is on parameter identification, the typical choice is to minimize some norm of the parameter feasible set $D_\theta(T_m)$. l_2 and l_∞ norms of the parameter uncertainty are the common choices. Note once again that l_∞ norm mainly consists in characterizing parameter uncertainties with tolerances while minimizing the l_2 norm is referred to as D-optimization. Here the l_∞ norm is used.
- When the focus is on the use of the estimated polynomial to predict the real one, the typical choice is to minimize some norm of the prediction error $E_{pred}(T_m)$ that is the difference between the estimated and the real (unknown) polynomial. Also in this case the l_∞ norm of the prediction error, that represents the maximum prediction error over the interval $[t_a, t_b]$, is a common choice. Such error is then given by

$$E_{pred}(T_m) = \sup_{\theta \in D_\theta(T_m)} \|P(t, \theta^o) - P(t, \theta)\|_\infty, \quad t \in [t_a, t_b] \quad (9)$$

Since $D_\theta(T_m)$ is a function of the measurement error realization, consequently also its norm, that accounts for estimates uncertainties, and the prediction error, as defined in (9), depend on the error realization. To get rid of this and in order to provide guaranteed results it is common practice to look for the worst case estimation uncertainties and worst case prediction error that are the

maximal values that uncertainties and error can assume for any possible measurement error realization. This is achieved considering the norm of set $\tilde{D}_\theta(T_m)$ instead of the one of set $D_\theta(T_m)$ and similarly considering the worst case prediction error $\tilde{E}_{pred}(T_m)$ defined as

$$\tilde{E}_{pred}(T_m) = \sup_{\theta \in \tilde{D}_\theta(T_m)} \|P(t, \theta)\|_\infty, \quad t \in [t_a, t_b]. \quad (10)$$

2.2 Worst case estimation uncertainty in l_∞ norm

In the following some important properties of worst case estimation uncertainties are reported omitting technical details for the general case of linear in the parameter functions and for the particular case of monodimensional polynomials reported in relation (1). For a deeper understanding and more technicalities the interested reader should refer to papers [9], [10], [11] and the references therein.

- Worst case estimation uncertainties affecting parameter estimates can be computed for any given sampling schedule $T_m = \{\tau_1, \tau_2, \tau_3 \dots \tau_m\}$. Worst case estimation analysis can be performed without collecting any real measurement.
- Let $T_o(T_m)$ be the set of minimum cardinality among all the possible subsets of T_m that ensure the same worst case estimation uncertainties that are obtained using the set T_m . $T_o(T_m)$ is referred to as the optimal sampling set. Its optimality consists in the fact that it is the set that ensures the same worst case estimation uncertainty of T_m with the minimum number of measurements.
- The cardinality of $T_o(T_m)$ is always bounded between p and p^2 (the number of parameters and its square).
- In general $T_o(T_m)$ is not unique.
- The set $T_o(T_m)$ for linear in the parameters functions can be numerically derived solving p linear programming problems as described in [8]. Remark, however, that such numerical procedure could lead to a suboptimal sampling set $T_{so}(T_m)$ whose cardinality is still between p and p^2 , but it results to be $T_{so}(T_m) \supset T_o(T_m)$. Furthermore the numerical procedure does not allow to recognize possible multiple optimal sampling sets.
- For monodimensional polynomials represented in the Vandermonde base as in relation (1) the optimal sampling schedule is unique and consists of only p elements.
- Define T_∞ as the set of infinite cardinality consisting of all sampling times in the interval $[t_a, t_b]$. Its corresponding optimal sampling schedule $T_o(T_\infty)$ can be analytically derived and is constituted by the points in which the 1st kind Chebyshev polynomial of degree $p-1$ associated with the interval $[t_a, t_b]$ achieves its minimal and maximal values. Such sampling schedule will be referred to as the optimal estimation error sampling schedule.
- If the parameter estimation is performed using measurements collected according to a sampling schedule T_p of cardinality p , then the actual

estimation uncertainties are equal to the worst case estimation uncertainties.

Remark that worst case uncertainty is computed a priori assuming the worst possible error realization and dealing with set $\tilde{D}_\theta(T_m)$. Once actual measurements are collected it is possible to compute the actual estimate uncertainties that affect the parameter estimates and depend on the error realization operating on set $D_\theta(T_m)$. Hereafter some properties of actual estimate uncertainties are reported together with their relation with the worst case parameter uncertainties.

- If the parameter estimation is performed using measurements collected according to a sampling schedule T_m of cardinality $m > p$, in general the actual estimation uncertainties can be smaller than the worst case estimation uncertainties.
- Let T_m (or T_∞) be the sampling schedule collecting all possible measurements, let $T_o(T_m) \subset T_m$ and say r the cardinality of $T_o(T_m)$. If more than r measurements can be actually collected, under mild statistical assumption on the measurement error distribution, the best choice consists in measuring several time the function in the same sampling times of the optimal sampling $T_o(T_m)$ [12].

2.3 Worst case prediction error

For what concerns the worst case prediction error $\tilde{E}_{pred}(T_m)$, the case that has been extensively studied is the one of monodimensional polynomials of degree $p-1$, when only p sampling points in the finite interval $[t_a, t_b]$ have to be used. According to a conjecture by Bernstein [13] which has been proved by Kilgore [14] and De Boor – Pinkus [15] the minimum worst case prediction error function is piecewise polynomial that has one local maximum on each interval $[t_i, t_{i+1}]$ (with $t_{i+1} > t_i$, $i = 1, \dots, p-1$). The sampling schedule that minimizes the worst case prediction error over the interval $[t_a, t_b]$ is the one for which the local maxima on each interval $[t_i, t_{i+1}]$ are equal. Unfortunately no closed form is available for such schedule which can be only numerically derived. However the schedule is very well approximated by the roots of the 1st kind Chebyshev polynomial of degree p translated on the interval $[t_a, t_b]$. Such sampling schedule will be referred to as the optimal prediction error sampling schedule.

Indeed when a sampling schedule T_m is assigned, it is quite easy to derive numerically the worst case prediction error over the interval $[t_a, t_b]$.

Table 1: Worst case prediction errors with different sampling schemes for polynomial of different degree

| | Polynomial Degree | | | | |
|--------------------------------|-------------------|------|------|------|------|
| | 2 | 3 | 4 | 5 | 6 |
| Opt. prediction error sampling | 1.25 | 1.43 | 1.57 | 1.68 | 1.78 |
| Opt. estimation sampling | 1.25 | 1.67 | 1.80 | 1.99 | 2.08 |

For example and for future reference in Table 1 are reported the worst case prediction errors that can be achieved using the optimal prediction error sampling schedule as well as the optimal estimation error sampling

schedule. Results are for polynomials of different degree and are obtained normalizing the error bound $E = 1$.

The case in which m measurement points over the interval $[t_a, t_b]$ can be used, to the best of our knowledge, has not been studied when $m > p$. It is however easy to state that also in this case the minimum worst case prediction error function is piecewise polynomial and to conjecture that it has a local maximum on each interval $[t_i, t_{i+1}]$ (with $t_{i+1} > t_i$, $i = 1, \dots, m$). Also in this case holds the conjecture that the sampling schedule that minimizes the worst case prediction error over the interval $[t_a, t_b]$ is the one for which the local maxima on each interval $[t_i, t_{i+1}]$ are equal. In this case the optimal schedule has to be numerically derived. The conjecture that the schedule could be well approximated by the roots of the 1st kind Chebyshev polynomial of degree m translated on the interval $[t_a, t_b]$ has been proven false. In Table 2 the worst case prediction error which results when using a uniform schedule, the schedule provided by the roots of the 1st kind Chebyshev polynomials, and numerically evaluated almost optimal schedules is reported for different polynomial degrees and for different number of measurements in $[t_a, t_b]$. Numerical results are computed normalizing to one the bound on the error so that $E = 1$. It can be noted that the schedule provided by the roots of the 1st kind Chebyshev polynomials is not optimal when the number of measurements is larger than the degree of the identified polynomial. Moreover its corresponding worst case prediction error is not monotonically decreasing for increasing number of sampling points in $[t_a, t_b]$.

3 OPTIMAL SAMPLING SELECTION FOR MULTIDIMENSIONAL POLYNOMIALS THAT ARE THE SUM OF MONODIMENSIONAL ONES

Very few results are available for the optimal sampling selection of multidimensional polynomials. In [16] the optimal sampling schedule for worst case estimation uncertainty is derived for n -dimensional polynomials whose monomials are the Cartesian product of the monomials of n distinct monodimensional polynomials.

For other multidimensional polynomials only the numerical techniques described in [9] can be applied.

As discussed in the introduction, the case in which n -dimensional polynomials are the sum of n monodimensional ones, each one of which is function of a different variable, is of particular interest. In this section attention is focused on this particular class of polynomials for which some important results are reported. In order to simplify the notation, results are presented for 2-dimensional polynomials of the form

$$P(x, y) = \theta_0 + \theta_{1x}x + \theta_{2x}x^2 + \dots + \theta_{lx}x^l + \theta_{1y}y + \theta_{2y}y^2 + \dots + \theta_{ly}y^l \quad (4)$$

in which variables x and y are bound in the intervals $[x_a, x_b]$ and $[y_a, y_b]$ respectively. Extension to polynomials of higher dimension is straight forward.

3.1 Worst case estimation uncertainty

Let $T_{\infty, \infty} = \{[x_a, x_b] \times [y_a, y_b]\}$ be the set of all possible measurement where the polynomial of relation (4) could be measured. In order to get the optimal sampling schedule $T_o(T_{\infty, \infty})$ no closed form solution is available and only a numerical solution can be sought applying the procedure described in [9] to a finite dimension set $\tilde{T}_{\infty, \infty}$ that approximates $T_{\infty, \infty}$ and is constituted by the points on a tight grid on $[x_a, x_b] \times [y_a, y_b]$. The tighter the grid the higher the cardinality of $\tilde{T}_{\infty, \infty}$ and the better its approximation of $T_{\infty, \infty}$. However very high cardinality of $\tilde{T}_{\infty, \infty}$ leads to computational and numerical problems. The problem can be simplified making use of the following result that is stated as a conjecture since it is supported by numerical evidence, but has not been proved. To this extent consider the sampling schedule $T_{l,j}$ whose elements are the nodes of the grid that is obtained using in the x dimension the optimal estimation uncertainty sampling schedule for l -th order polynomials and in the y dimension the optimal estimation uncertainty sampling schedule for j -th order polynomials.

Table 2: worst case prediction error which results when using a uniform schedule, the schedule provided by the roots of the 1st kind Chebyshev polynomial, and a numerically evaluated almost optimal schedule

| Polynomial degree | | Number of measurements used for identification | | | | | | |
|-------------------|------------------------------|--|------|------|------|------|------|------|
| | | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | Uniform sampling schedule | 1.25 | 1.08 | 1.08 | 1.04 | 1.04 | 1.03 | 1.02 |
| | Chebyshev sampling schedule | 1.41 | 1.12 | 1.15 | 1.07 | 1.08 | 1.05 | 1.05 |
| | Numerically optimal schedule | 1.25 | 1.08 | 1.08 | 1.04 | 1.04 | 1.03 | 1.02 |
| 3 | Uniform sampling schedule | 1.63 | 1.29 | 1.19 | 1.19 | 1.11 | 1.08 | 1.06 |
| | Chebyshev sampling schedule | 1.43 | 1.37 | 1.41 | 1.17 | 1.15 | 1.16 | 1.09 |
| | Numerically optimal schedule | - | 1.25 | 1.16 | 1.12 | 1.08 | 1.06 | 1.05 |
| 4 | Uniform sampling schedule | - | 2.21 | 1.71 | 1.41 | 1.29 | 1.22 | 1.17 |
| | Chebyshev sampling schedule | - | 1.57 | 1.66 | 1.40 | 1.41 | 1.19 | 1.21 |
| | Numerically optimal schedule | - | - | 1.39 | 1.25 | 1.19 | 1.15 | 1.13 |
| 5 | Uniform sampling schedule | - | - | 3.10 | 2.18 | 1.88 | 1.59 | 1.43 |
| | Chebyshev sampling schedule | - | - | 1.68 | 1.63 | 1.48 | 1.39 | 1.41 |
| | Numerically optimal schedule | - | - | - | 1.47 | 1.36 | 1.25 | 1.20 |
| 6 | Uniform sampling schedule | - | - | - | 4.55 | 3.09 | 2.30 | 2.05 |
| | Chebyshev sampling schedule | - | - | - | 1.78 | 1.84 | 1.67 | 1.51 |
| | Numerically optimal schedule | - | - | - | - | 1.56 | 1.44 | 1.35 |

This grid is referred to as optimal estimation uncertainty grid.

Conjecture: *The optimal sampling schedule $T_o(T_{l,j})$ computed for the sampling schedule $T_{l,j}$ provided by the optimal estimation uncertainty grid is equal to the optimal sampling schedule $T_o(T_{\infty,\infty})$ of the $T_{\infty,\infty}$ set.*

The conjecture appears to state a strong result since it allows to reduce the search of the optimal sampling schedule from a set of dimension ∞^2 to a set of dimension $(l+1) \cdot (j+1)$. This indeed is even more important for n -dimensional polynomials, when the reduction is from an ∞^n dimensional set. However in practice the result is not as strong as it looks since the following facts hold.

- The numerical procedure, described in [9], to get the optimal sampling schedule $T_o(T_{l,j})$ usually leads to a suboptimal sampling set $T_{so}(T_{l,j})$.
- In general there are more than one optimal sampling schedule $T_o(T_{l,j})$.
- In general the optimal sampling schedule changes if the domain $[x_a, x_b] \times [y_a, y_b]$ is shifted.

In the two dimensional case we are considering, as long as the degrees l and j are not too high, an exhaustive search of the optimal sampling schedule can be carried on. This was done for the particular case in which $l=j=5$ and normalizing $[x_a, x_b] \times [y_a, y_b] = [0,1] \times [0,1]$. In these conditions 36 possible sampling points have to be considered. It results that there are two optimal sampling sets of cardinality $p = l + j + 1 = 11$.

If instead $[x_a, x_b] \times [y_a, y_b] = [1,2] \times [1,2]$ is considered, no optimal sampling set of cardinality $p = l + j + 1 = 11$ can be found. This indeed can be explained with the fact that the shift of coordinates behaves like a change of base in the description of the polynomial and it is known that optimal sampling is sensitive to changes of base [11].

3.2 Worst case prediction error

For what concerns the worst case prediction error of the two dimensional polynomial in relation (4), a minimum of $m = l + j + 1 = 11$ measurements are needed. They should be collected $l+1$ different values of x and at $j+1$ different values of y in order to allow the parameter vector to be identifiable.

With such minimum number of measurements the prediction error is anyway rather consistent.

As an example considering again the particular case in which $l = j = 5$, $[x_a, x_b] \times [y_a, y_b] = [0,1] \times [0,1]$ and the error measurement error bound is $E = 1$, when using the two optimal sampling sets that ensure the minimum worst case parameter uncertainty the corresponding prediction error is equal to 9.00 in one case and 8.44. This indeed shows that the minimum number of measurements is not good enough to ensure a limited prediction error.

To ensure a limited prediction error the polynomial (4) should then be evaluated on a suitable grid of points out of $T_{\infty,\infty} = \{[x_a, x_b] \times [y_a, y_b]\}$. In this regard the optimal solution seems to be a sampling schedule T_{pred} whose elements are the nodes of the grid that is obtained using in the x dimension the optimal prediction error sampling schedule for l -th order polynomials and in the y dimension the optimal prediction error sampling schedule for j -th order polynomials. This grid is referred to as optimal prediction error uncertainty grid.

In the previously considered numerical case the use of this grid ensured a worst case prediction error of 1.68 that rises to 1.99 if the optimal estimation uncertainty grid $T_{l,j}$ is used, and further rises to 3.11 if a uniform 6×6 grid is used.

It is worth noting that while a grid on the whole two dimensional space is needed in order to ensure a limited worst case prediction error computed according to equation (10), once actual measurements are collected the actual prediction error is given by relation (9). Under mild assumption on the measurement error distribution it results that the actual prediction error is smaller than the worst case one.

4 CONCLUSIONS

In this paper the problem of sampling selection to identify multidimensional polynomials which are the sum of n monodimensional ones each one of which is the function of a different variable has been considered. Two optimal sampling selection schedules have been considered. The one aims to minimize the worst case uncertainty on the parameter estimates while the other aims to minimize the worst case prediction error that can be committed while using the identified polynomial to approximate the real unknown one.

While for the monodimensional case, when the optimal estimation error sampling (that minimizes the worst case estimate uncertainty) is used then also the resulting prediction error is relatively close to its minimum achievable value, for the multidimensional case this is no longer true. On top of this it appears that in general the optimal estimation error sampling in the multidimensional case is hard to be found since numerical algorithms which are the only available tool, often provide suboptimal solutions.

In the multidimensional case when the prediction error is of concern and it must be reduced to a minimum, measurements need to be collected on the nodes of a grid in the n -dimensional space that is obtained selecting, in each dimension, the monodimensional optimal prediction error sampling schedule.

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